

# Optimal Extraction of Cosmological Information from Supernova Data in the Presence of Calibration Uncertainties

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We present a new technique to extract the cosmological information from high-redshift supernova data in the presence of calibration errors and extinction due to dust. While in the traditional technique the distance modulus of each supernova is determined separately, in our approach we determine all distance moduli at once, in a process that achieves a significant degree of self-calibration. The result is a much reduced sensitivity of the cosmological parameters to the calibration uncertainties. As an example, for a strawman mission similar to that outlined in the SNAP satellite proposal, the increased precision obtained with the new approach is roughly equivalent to a factor of five decrease in the calibration uncertainty.

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## I. INTRODUCTION

The study of type-Ia supernovae provided the first indication that the expansion of the Universe is accelerating [1, 2]. This conclusion is supported by the combination of Cosmic Microwave Background results [3] and large-scale structure measurements [4]. Results from more recent supernova surveys [5, 6] further strengthen the evidence for the accelerated expansion of the Universe and the necessity for some mysterious mechanism that drives it, code-named “dark energy.”

Supernova magnitudes at peak brightness and redshifts are the ingredients that allow their use as cosmological probes. Peak magnitudes can be corrected with the “stretch” [1],  $\Delta m_{15}$  [7], or MLCS [8] techniques to turn type-Ia supernovae into standardized candles to a level around 0.10–0.15 mag (10–15% in flux). Therefore, measuring the supernova apparent magnitude provides the distance and hence the lookback time to when the explosion occurred, while measuring its redshift  $z$  provides the scale factor  $a = (1+z)^{-1}$  at the time the supernova light was emitted. These two statistics for each supernova from a set that spans a range of redshifts provide a mapping of the cosmic expansion with time.

Neglecting for the moment any secondary effects, one can relate the apparent peak magnitude  $m(z)$  of a supernova at redshift  $z$  to the luminosity distance  $d(z) = r(z)(1+z)$ , where  $r(z)$  is the dimensionless comoving distance, through:

$$m(z) \equiv -2.5 \log_{10} (\mathcal{F}/\mathcal{F}_0) = \mathcal{M} + \mu(z) = \mathcal{M} + 5 \log_{10} d(z) + 25 \quad (1)$$

where  $\mathcal{F}$  is the observed supernova flux and  $\mathcal{F}_0$  is the flux corresponding to zero magnitude of the magnitude system.  $\mathcal{M} \equiv M - 5 \log_{10} [H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}]$ , can be regarded as an unknown nuisance constant,  $M$  being the absolute supernova magnitude, while  $\mu$  is the dimensionless supernova distance modulus. With this definition  $r(z)$  is also dimensionless and does not depend on  $H_0$ :

$$r(z) = \int_0^z dz' H_0 \frac{a}{\dot{a}} = \int_0^z dz' \frac{H_0}{H(z')} , \quad (2)$$

$$(H(z)/H_0)^2 = \Omega_m (1+z)^3 + (1-\Omega_m) \exp \left[ 3 \int_0^z \frac{dz'}{1+z'} (1+w(z')) \right] ,$$

where we have assumed a flat universe and  $w(z)$  is the equation of state of the dark energy,  $w(z) = -1$  for a cosmological constant. In more general models  $w(z)$  would simply be a function that captures the effect of dark energy. Following [9, 10] we consider the parameterization

$$w(z) = w_0 + w_a (1 - a(z)) , \quad (3)$$

where  $w_0$  and  $w_a$  are constants that determine, respectively, the dark-energy equation of state now and its evolution.

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In a real observation there are many more effects that have to be accounted for: weak lensing, Milky Way extinction, etc. In this paper, we will concentrate on two (inter-related) effects: host-galaxy extinction due to dust and mis-calibration. Some of the other effects are treated in [11]. Dust in the supernova host galaxy will dim the light coming from the supernova, therefore biasing its measured magnitude toward larger values. However, ordinary dust dims light of different wavelengths differently, absorbing more blue light than red light. By measuring the supernova flux in several broadband filters, one can determine the reddening and hence the overall dimming that dust creates. In order to proceed with the last step two things are needed: a good understanding of the gross features of non-extincted type-Ia supernova spectra, and a knowledge of the relationship between differential dimming across wavelengths and absolute extinction. We will assume a perfect knowledge of the former and for the latter we will use the standard Cardelli, Clayton and Mathis model [12]. In this model, the extinction  $A$  (that is, the dimming in magnitude units) at a wavelength  $\lambda$  can be written as:

$$A(\lambda) = A_V \cdot (a(\lambda) + b(\lambda)/R_V) \quad (4)$$

where the parameters  $A_V$  and  $R_V$  control, respectively, the overall amount of dimming and its dependence on wavelength, and  $a(\lambda)$  and  $b(\lambda)$  are known functions. The value of  $A_V$  will be different for different supernovae depending on the properties of the host galaxy and the position of the supernova inside it. The value of  $R_V$  will depend on the type of dust (grain size, for instance), and observations show [13] that it varies significantly across redshift and between galaxies. In the following, we will assume no prior knowledge on either  $A_V$  or  $R_V$ . Then, given enough independent measurements of the magnitudes of a given supernova  $i$  in several broadband filters centered around wavelengths  $\lambda_\alpha$ , one can in principle determine directly from the observations the dust parameters  $A_V^i$ , and  $R_V^i$  for supernova  $i$ , together with its distance modulus.

Filter calibration uncertainties affect the measured magnitude in each passband. Therefore, they affect not only the overall peak magnitude but also the extinction correction that, as mentioned above, depends on understanding the supernova colors (defined as the ratio of the supernova flux in two different filters). The calibration error in a given observer filter will affect each supernova in a different (but correlated) way, since a given passband in the supernova rest frame will be observed with a different filter in the observatory depending on the supernova redshift. Calibration uncertainties are going to be among the limiting uncertainties [11] in the new generation of supernova surveys being currently planned.

In the standard way of extracting cosmological information from supernova data, the data for a given supernova  $i$  is used to determine the values of  $\mu^i$ ,  $A_V^i$ , and  $R_V^i$  and their uncertainties. Then the calibration error is propagated through that supernova's measurements. As a result, a non-diagonal covariance matrix for the set of distance moduli  $\{\mu^i\}$  is obtained. The measurements of all the  $\{\mu^i\}$  and their covariance matrix are then used in a fit to determine the cosmological parameters  $\Omega_m$ ,  $w_0$ , and  $w_a$ , using Eqs. (1), (2) and (3).

In our approach, all  $\{\mu^i\}$  are determined simultaneously. In this way, all information available is used in an optimal way, which achieves a certain degree of self-calibration. The cosmological parameter fit then follows as above. Equivalently, one can just bypass completely the step of extracting the distance moduli and determine directly the cosmological parameters from the set of magnitudes measured for all supernovae  $\{m_j^i\}$  ( $i$  runs over supernovae, whereas  $j$  runs over passbands in the supernova rest frame). In Section II we present a fiducial supernova mission: the dataset it provides and the algorithms through which the dataset is used to measure the dark energy. Within that framework, we elaborate further on the calibration error and its interplay with extinction. Section III describes both the traditional data analysis method and our new approach. Results using both approaches are given in Section IV for a fiducial mission and the statistical advantage that the new, optimal method affords becomes apparent. Finally, we draw our conclusions in Section V.

## II. CALIBRATION AND SUPERNOVA COSMOLOGY

Although our results have a more general validity, in the remainder we discuss a specific strawman mission along the lines of the SNAP satellite proposal [14]. The mission observes 2000 type-Ia supernovae over a redshift range from 0 up to 1.7 with a set of nine logarithmically-spaced broadband filters centered at wavelengths  $\lambda_\alpha = \lambda_0 \cdot (1 + 0.16)^\alpha$ ,  $\alpha = 0, \dots, 8$ , with  $\lambda_0 = 440$  nm corresponding to the  $B$  band. For every supernova, the analysis is based on the  $B$  passband in the supernova rest frame and any other band redder than this which, after redshifting to the observer frame, still ends up overlapping one of the nine filters. So only optical and infrared information in the supernova rest frame is used. The current lack of understanding of supernova heterogeneity in the UV prevents the robust usage of shorter wavelengths (though see [15, 16] for perspectives for using UV light curves to aid in measuring supernova distances).

Therefore,  $9 - k$  filters are used for a supernova with  $z$  between  $z_k$  and  $z_{k+1}$ , where  $z_k = (1 + 0.16)^k - 1$ ,  $k = 0, \dots, 6$ . With this choice of filters, at least three bands are available for each supernova. The calibration uncertainty enters

through the zero points of the filters,  $\mathcal{Z}_\alpha = 2.5 \log_{10} \mathcal{F}_{\alpha 0}$ ,  $\alpha = 0, \dots, 8$ , where  $\mathcal{F}_{\alpha 0}$  is the observed flux measured in filter  $\alpha$  for a standard object of known magnitude 0. So in order to specify a calibration error, one has to specify the  $9 \times 9$  covariance matrix for  $\{\mathcal{Z}_\alpha\}$ .

We are now in a position to write the equation that will relate the measured light-curve-shape-corrected [1, 7, 8] and K-corrected [17] peak magnitudes  $m_j^i$  with the parameters of our model (details on K-correction uncertainties, which we will not consider here, can be found in [18]):

$$m_j^i + \mathcal{Z}_{\alpha(i,j)} = \mathcal{M}_j + \mu^i(z^i; \Omega_m, w_0, w_a) + A_V^i \cdot a(\lambda_j) + B_V^i \cdot b(\lambda_j). \quad (5)$$

Indices  $i$  and  $j$  run respectively over supernovae and passbands.  $\mathcal{M}_j$  is  $\mathcal{M}$  for passband  $j$ :  $\mathcal{M}_j \equiv M_j - 5 \log_{10} [H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}]$ , where  $M_j$  is the absolute magnitude of a type-Ia supernova measured in passband  $j$ .  $B_V$  is defined as  $A_V/R_V$  and makes the problem linear on the dust parameters. Finally,  $\alpha(i, j)$  is the index of the filter in which, after K-correction, rest frame passband  $j$  of supernova  $i$  is measured. For  $z_k \leq z^i < z_{k+1}$ , we have  $\alpha(i, j) = j + k$ , and the index  $j$  runs from 0 to  $9 - k - 1$ , so that there are  $9 - k$  measurements for supernova  $i$ .

The left-hand side of Eq. (5) contains the measurements, while in the right-hand side we have the parameters to be determined:  $\mathcal{M}_j$ ,  $j = 0, \dots, 8$ , and  $\mu^i$ ,  $A_V^i$ ,  $B_V^i$  for each supernova. Alternatively, the fit parameters  $\mu^i$  can be replaced with the fit parameters  $\Omega_m$ ,  $w_0$ , and  $w_a$  with  $\mu^i \rightarrow \mu(z^i; \Omega_m, w_0, w_a)$  where the latter is the predicted magnitude as a function of the cosmological parameters. A careful examination of Eq. (5) reveals that there are two non-independent parameters. Specifically, a simultaneous shift in all  $A_V^i$  to  $A_V^i + \delta$ , can be compensated by a shift of all  $\mathcal{M}_j$  to  $\mathcal{M}_j - \delta \cdot a(\lambda_j)$ . Analogously, a change  $B_V^i \rightarrow B_V^i + \eta$  is exactly compensated by the shift  $\mathcal{M}_j \rightarrow \mathcal{M}_j - \eta \cdot b(\lambda_j)$ . Therefore, one has to eliminate two parameters. To do so, we assume that for a nearby supernova we know that it has no dust (for instance because it lies in an elliptical galaxy), or equivalently that its dust extinction is perfectly known. For our analysis, we choose to assume that for the supernova with index  $i = 0$  it holds  $A_V^0 = B_V^0 = 0$ . Furthermore, for simplicity, we assume that we know the peak magnitudes for that supernova,  $m_j^0$ , perfectly well. Results do not depend strongly on this assumption, which can be justified because the statistical errors in the photometry of a nearby supernova are going to be much smaller than those for the high-redshift ones. We can now subtract from Eq. (5) the particular equation for  $i = 0$ , to get

$$m_j^i - m_j^0 + \mathcal{Z}_{\alpha(i,j)} - \mathcal{Z}_j = \mu^i(z^i; \Omega_m, w_0, w_a) - \mu^0 + A_V^i \cdot a(\lambda_j) + B_V^i \cdot b(\lambda_j), \quad (6)$$

where we have taken into account  $\mathcal{Z}_{\alpha(0,j)} = \mathcal{Z}_j$  when supernova 0 has a low redshift  $z^0 < z_1 = 0.16$ . While the  $m_j^0$  magnitudes are assumed known,  $\mu^0$  is not and it becomes an additional free parameter. Now the index  $i$  runs from one to the number of supernovae minus one. The number of free parameters has been decreased by two with respect to Eq. (5). Written in this form, it also becomes clear that only eight color calibrations ( $\mathcal{Z}_\alpha - \mathcal{Z}_\beta$ ) rather than nine absolute calibrations are necessary for the cosmology determination. This is the framework that we use in the following.

### III. SIMULATED DATA ANALYSIS

#### A. Supernova-by-supernova procedure

In standard supernova-cosmology analysis, the distance to each supernova is determined independently. In our implementation of the traditional analysis procedure, values for  $A_V^i$ ,  $B_V^i$ , and  $\mu^i - \mu^0$  are determined from the set of magnitudes  $\{m_j^i\}$  for supernova  $i$ . Only statistical errors  $\sigma_{j, \text{stat}}^i$  in the measurements of the magnitudes are included in the fit. A Monte Carlo process is then used to compute the additional covariance matrix for the set of  $\{\mu^i\}$  due to the zero-point calibration uncertainties  $\sigma_{\alpha, \text{cal}}$ , where the index  $\alpha$  labels the filters. Finally, the cosmology fit is performed on all  $\{\mu^i\}$  using the covariance matrix that includes calibration errors plus an additional intrinsic dispersion  $\sigma_{\text{int}}^i$  for each supernova distance modulus that takes into account known supernova-to-supernova variability. The free parameters in the cosmology fit are  $\mu^0$ ,  $\Omega_m$ ,  $w_0$ , and  $w_a$ . For the fit we use the standard package `minuit`<sup>1</sup>. We have checked that a Fisher matrix calculation provides compatible error estimates.

<sup>1</sup> <http://wwwinfo.cern.ch/asdoc/minuit/minmain.html>

## B. Simultaneous procedure

In the new approach, all supernova data are used at once in order to determine all  $A_V^i$ ,  $B_V^i$ , and  $\mu^i - \mu^0$ . The input covariance matrix for the  $m_j^i$  includes statistical errors (which are diagonal), calibration error and intrinsic dispersion. For a large simulated data set of about 2000 supernovae, this treatment is rather impractical, since the input covariance matrix is about  $12000 \times 12000$  and non-diagonal, which makes it difficult to store and invert efficiently. Instead we can treat both the intrinsic dispersion and the calibration error as new free fit parameters, constrained within the known uncertainties. Then one can build a  $\chi^2$  function whose covariance matrix is diagonal:

$$\chi^2 = \sum_{ij} \left( \frac{m_j^i - m_j^0 - \bar{m}_j^i(\mu^i - \mu^0, A_V^i, B_V^i, S^i, \mathcal{Z}_\alpha)}{\sigma_{j, \text{stat}}^i} \right)^2 + \sum_i \left( \frac{S^i}{\sigma_{\text{int}}^i} \right)^2 + \sum_\alpha \left( \frac{\mathcal{Z}_\alpha}{\sigma_{\alpha, \text{cal}}} \right)^2, \quad (7)$$

$$\bar{m}_j^i = \mu^i(z^i; \Omega_m, w_0, w_a) - \mu^0 + A_V^i \cdot a(\lambda_j) + B_V^i \cdot b(\lambda_j) + S^i - \mathcal{Z}_{\alpha(i,j)} + \mathcal{Z}_j, \quad (8)$$

where in Eq. (7), again the indices  $i$  and  $j$  run over supernovae and passbands respectively,  $\alpha$  runs over filters, and  $S^i$  is the new free parameter that accounts for the intrinsic supernova dispersion, one new parameter per supernova. Now the  $\{\mathcal{Z}_\alpha\}$  are considered as nine additional free parameters, constrained to vary within the calibration precision. The uncertainties  $\sigma_{j, \text{stat}}^i$ ,  $\sigma_{\text{int}}^i$ , and  $\sigma_{\alpha, \text{cal}}$  are as defined in Section III A. Typically, for 2000 supernovae, we now have about 12000 measurements and over 8000 unknowns. After all  $\{\mu^i - \mu^0\}$  are determined, the cosmology fit proceeds as in Section III A, but now using the  $\{\mu^i\}$  covariance matrix from the overall fit to all distance moduli.

Alternatively, and equivalently, one can do without the determination of the distance moduli, and just fit all the magnitudes directly to the cosmological parameters and the other nuisance parameters, by using again the  $\chi^2$  in Eqs. (7,8). Now the free parameters in the fit are all the  $A_V^i$ ,  $B_V^i$ ,  $S^i$  and  $\mathcal{Z}_\alpha$ , as well as  $\mu^0$ ,  $\Omega_m$ ,  $w_0$ , and  $w_a$ . This is the final approach that we have used. A Fisher matrix calculation following these lines runs on a 64-bit<sup>2</sup>, 3 GHz Intel Xeon processor in about one hour for 2000 supernovae.

## IV. RESULTS

### A. Calibration models

In this section we are going to present results based on two extreme calibration models. We will not attempt to construct a realistic model from observations of standard stars or calibrators. Instead, we will specify the calibration model by its  $9 \times 9$  zero-point covariance matrix.

In the first model, we assume that the zero-point covariance matrix is fully diagonal. While this is surely an unrealistic scenario (the same fundamental calibrator is probably going to be used for more than one filter, if not all), there will still be a diagonal component in the full calibration covariance matrix, originating for instance from Poisson uncertainties in the measurement of the fundamental calibrator. For simplicity, we will take the calibration errors in all filters to be equal, so that the zero-point covariance matrix will be simply  $V_{\alpha\beta} = \sigma_{\text{cal}}^2 \cdot \delta_{\alpha\beta}$ . We will call this model the “Diagonal” model.

The other model we consider goes to the opposite extreme, and assumes that all sources of error during the calibration process come from the uncertainty in a single parameter, such as the temperature of a hot white dwarf used as a calibrator. In this case the zero-point parameters are replaced with a single temperature parameter  $T$  and Eq. (7) has to be modified to replace the last term with a term  $(T - T_0)^2 / \sigma_T^2$  where  $T_0$  is the given temperature and  $\sigma_T$  the precision with which it is known. Then, in Eq. (8) the zero points  $\mathcal{Z}_\alpha$  are turned into functions of the temperature parameter by integrating in the corresponding filter the black body spectrum of the white dwarf with that temperature. We will call this model the “Temperature” model.

As mentioned above, these are two very extreme toy models for the calibration covariance matrix. The true covariance matrix will be much more complicated and will depend on the specific details of the mission and on the set of standard calibrators used. However, these two models should span the range of possible calibration error models.

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<sup>2</sup> Use of 128-bit (double) precision arithmetic is crucial in the process of inverting the about  $6000 \times 6000$  heavily-correlated Fisher matrix.

TABLE I: The redshift distribution  $N(z)$  of the supernovae employed in the fiducial survey. The redshifts  $z$  given in the table correspond to the value for all supernovae in that bin.

$z$	0.05	0.17	0.35	0.57	0.82	1.11	1.50
$N(z)$	317	82	219	412	441	427	400

TABLE II: Uncertainties in the cosmological parameter determination in the two analysis methods for several values of the calibration uncertainty in the “Diagonal” error model.

$\sigma_{\text{cal}}$		SN by SN	Simultaneous
0.000	$\sigma(w_0)$	0.064	0.064
	$\sigma(w_a)$	0.30	0.30
0.001	$\sigma(w_0)$	0.082	0.068
	$\sigma(w_a)$	0.40	0.33
0.005	$\sigma(w_0)$	0.099	0.071
	$\sigma(w_a)$	0.59	0.43
0.010	$\sigma(w_0)$	0.11	0.075
	$\sigma(w_a)$	0.81	0.53

### B. No calibration error

To start with, we consider the case with no calibration error. That is, we fix all parameters  $\{\mathcal{Z}_\alpha\}$  in Eq. (7) or equivalently, we take the limit  $\sigma_{\alpha, \text{cal}} \rightarrow 0$ . Let us now specify completely our fiducial survey. Table I gives the number of supernovae in each redshift bin, based on the numbers in [11]. All supernovae in a bin have been assigned the same actual redshift, which is also reported in the table. In total there are 2298 supernovae, 300 of which would be obtained at low redshift by ground-based observations like those of the Supernova Factory [19]. We consider that the calibration error in the observation of these 300 additional supernovae is completely correlated with the calibration error in the original 2000 supernovae. This is what will happen if the dominant calibration uncertainty comes from the limited knowledge of the intrinsic properties of the calibrators, and the same calibrators are used in both surveys.

For the statistical errors in the magnitudes we choose  $\sigma_{j, \text{stat}}^i = 0.01$  for all passbands in all supernovae. This uncertainty encompasses uncorrelated statistical uncertainties introduced from the photometry process (e.g. flatfield errors, Poisson noise) and intrinsic uncorrelated supernova color dispersion. The intrinsic dispersion in the supernova magnitude is  $\sigma_{\text{int}}^i = 0.15$  for all supernovae, independent from supernova to supernova and fully correlated for all passbands within one supernova. Other than the supernova measurements, we will include in our synthetic data sample a measurement of the distance to the surface of last scattering from the future Planck survey, with a relative precision of 0.7%. Finally, we take as our fiducial cosmology the flat  $\Lambda$ CDM model,  $w_0 = -1$ ,  $w_a = 0$ , with  $\Omega_m = 0.28$ .

The resulting uncertainties in the cosmological parameters  $w_0$  and  $w_a$  are given in the first two rows of Table II. When there are no calibration errors the results obtained with the two analysis methods fully agree, as they should, given that in this case there is no information shared between any two supernovae, and therefore it should not make a difference whether they are analyzed separately or all at once.

### C. Diagonal calibration model

For the “Diagonal” calibration error model, we will choose  $V_{\alpha\beta} = \sigma_{\text{cal}}^2 \cdot \delta_{\alpha\beta}$ , with  $\sigma_{\text{cal}}$  varying between 0.001 and 0.01. The results again are in Table II. Since now, because of the calibration error, there is correlated information shared among the supernovae, there is a clear advantage in analyzing all supernovae at once, to the point that, for instance, the effect of a 0.005 calibration uncertainty using the simultaneous fit is reduced to approximately the effect of a 0.001 calibration uncertainty when considering each supernova individually.

Inspection of the final uncertainties in  $\mathcal{Z}_\alpha$  after the Fisher matrix calculation reveals the amount of self-calibration achieved: for example when 0.010 is assumed as the prior error in each  $\mathcal{Z}_\alpha$ , the final (posterior)  $\mathcal{Z}_\alpha$  errors are reduced to values ranging between 0.004 and 0.008, with large positive correlations between neighboring filters.

#### D. One-parameter calibration model

In the “Temperature” model, one single parameter, in this case the temperature of a single  $T_0 = 20000\text{K}$  hot white dwarf, defines the whole calibration error matrix. In this case, the final precision in the cosmological parameters is rather insensitive to the calibration error as was found in [11]. For instance, assuming a temperature error such that it results in an uncertainty for the zero point of the first filter of  $\sigma_{0,\text{cal}} = 0.01$  (and totally correlated uncertainties in the other zeropoints) results in an inappreciable change in the error on  $w_0$  and  $w_a$ . Even if we assume  $\sigma_{0,\text{cal}} = 0.10$ , the uncertainties increase only marginally from 0.064 to 0.066 ( $w_0$ ) and from 0.30 to 0.31 ( $w_a$ ). These results hold when performing the analysis in either of the two methods. The reason is that with one single parameter defining the calibration error, self-calibration can occur already within a single supernova, provided there are at least four passbands available, which is the case for over 80% of our supernovae.

#### V. CONCLUSIONS

In summary, we have presented a new method of analysis of high-redshift supernova data in the presence of calibration uncertainties and dust extinction that performs the statistical analysis of all data at once in order to self-calibrate the zeropoints of the filter set. For a fiducial mission inspired on that specified in the SNAP proposal, a significant reduction of the sensitivity of the cosmological parameters to the calibration uncertainty is achieved with the new method, equivalent to a reduction of about a factor five in calibration uncertainty. The advantage of the simultaneous analysis is not only applicable to calibration uncertainties, but to all sources of uncertainty that are non-trivially correlated between supernovae.

Calibration uncertainties are no longer treated as irreducible; the supernova data provide an avenue for self-calibration of magnitude zeropoints. Assuming that type-Ia supernovae are standardizable candles in our passbands, their colors as a function of redshift can be predicted. Measurements of supernova colors at different redshifts therefore constrain the differences in zeropoints,  $\{\mathcal{Z}_\alpha - \mathcal{Z}_\beta\}$ . Some of the color information is spent in correcting for dust extinction of individual supernovae, however given enough supernovae observed in enough passbands, a significant improvement in calibration is still achieved.

This paradigm for simultaneous supernova analysis gives rise to new questions that can be addressed in future studies. The interplay between the supernova sample, input calibration uncertainty, cosmology priors, photometry uncertainty, and intrinsic supernova magnitude and color dispersion needs to be explored. The effects of merging disparate supernova samples whose calibrations are not perfectly correlated can be studied. Equally important is an exploration of systematic uncertainties incurred if incorrect models for either dust-extinction or intrinsic supernova colors are used. The generation of a realistic covariance matrix that faithfully describes the calibration process would show where between the “Diagonal” and “Temperature” models realistic observations will lie.

For large data sets like those in [14] the new method presents some practical numerical problems. In this study, a Fisher matrix analysis has been performed, which provides reliable estimates of the attainable precision. In the future, a Monte Carlo approach should allow for a real fit to the data, as well as facilitate the inclusion of non-Gaussian priors in, for instance, the distributions of  $A_V^i$  and  $R_V^i$ .

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